

The COMPLEIK subroutine of the IONORT-ISP system
for calculating the non-deviative absorption:
A comparison with the ICEPAC formula

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Abstract

The present paper proposes to discuss the ionospheric absorption, assuming a quasi-flat layered ionospheric medium, with small horizontal gradients. A recent complex eikonal model [Settimi et al., 2013b] is applied, useful to calculate the absorption due to the ionospheric D-layer, which can be approximately characterized by a linearized analytical profile of complex refractive index, covering a short range of heights between $h_1 = 50$ km and $h_2 = 90$ km. Moreover, Settimi et al. [2013c] have already compared the complex eikonal model for the D-layer with the analytical Chapman's profile of ionospheric electron density; the corresponding absorption coefficient is more accurate than Rawer's theory [1976] in the range of middle critical frequencies. Finally, in this paper, the simple complex eikonal equations, in quasi-longitudinal (QL) approximation, for calculating the non-deviative absorption coefficient due to the propagation across the D-layer are encoded into a so called COMPLEIK (COMPLEx EIKonal) subroutine of the IONORT (IONOspheric Ray-Tracing) program [Azzarone et al., 2012]. The IONORT program, which simulates the three-dimensional (3-D) ray-tracing for high frequencies (HF) waves in the ionosphere, runs on the assimilative ISP (IRI-SIRMUP-P) discrete model over the Mediterranean area [Pezzopane et al., 2011]. As main outcome of the paper, the simple COMPLEIK algorithm is compared to the more elaborate semi-empirical ICEPAC formula [Stewart, undated], which refers to various phenomenological parameters such as the critical frequency of E-layer. COMPLEIK is reliable just like the ICEPAC, with the advantage of being implemented more directly. Indeed, the complex eikonal model depends just on some parameters of the electron density profile, which are numerically calculable, such as the maximum height.

Keywords: Ionospheric D-layer, Quasi-Longitudinal propagation, non-deviative absorption, ICEPAC formula, ISP-IONORT, Complex Eikonal theory.

1. Introductory review

Absorption is the process by which ordered energy of the radio wave is transformed into heat and electromagnetic (e.m.) noise by electron collisions with neutral molecules and ionized particles.

1.1 Collisions

Collisions of free electrons with neutrals, heavy ions or other electrons are important for various macroscopic phenomena. At higher altitudes (E-region), they determine the thermal and electrical conductivity of the plasma and thus the current systems which give rise to geomagnetic variations. Another important role of electron collisions is the absorption of radio waves which occurs at lower altitudes (D-region). Conversely, the collision frequency can be deduced from radio wave propagation data with an appropriate theory [Davies, 1990].

The collision cross-section σ for electrons in the nitrogen N_2 — the most abundant atmospheric constituent up to 100 km — was measured by Phelps and Pack [1959] and found to be proportional to the root-mean-square electron velocity v_{rms} . In consequence, Sen and Wyller [1960] generalized the Appleton-Hartree magneto-ionic theory to: (a) include a Maxwellian velocity distribution of the electrons, and (b) extend the findings of Phelps and Pack [1959] to all constituents of air, i.e. $\sigma \sim v_{\text{rms}}$. A further consequence is that the momentum transfer collision frequency ν of electrons with thermal energy $k_B T$ becomes proportional to pressure p (being k_B the Boltzmann's constant and T the absolute temperature).

The atmosphere in the E and D layers mainly consists of nitrogen N_2 (about 80%), with atomic and molecular oxygen O_2 as the next most important constituents. The relatively large cross section for N_2 makes it likely that, as first-order approximation, the height variation of collision frequency ν is proportional to the partial pressure of the N_2 present. Experiments show that the cross

section for O_2 also varies with the square root of T so that the two contributions can be combined [Thrane and Piggott, 1966].

When there is complete mixing of the atmosphere gases [Budden, 1988]:

$$\frac{dp}{p} = \frac{d\rho_N}{\rho_N} + \frac{dT}{T} = -\frac{dH}{H}, \quad (1.a)$$

where p is the total pressure, ρ_N the number density, T the absolute temperature of molecules and $H = k_B T / mg$ the atmospheric scale height, being g the gravity acceleration and m the mean molecular mass.

Just for this reason, the collision frequency ν varies with the height h above ground as:

$$\nu(h) = \nu_0 \frac{p(h)}{p_0} = \nu_0 \exp\left(-\frac{h-h_0}{H}\right). \quad (1.b)$$

According to Budden [1965], the generalized theory [Sen and Wyller, 1960] is important in the detailed quantitative interpretation of certain experiments, but, for most practical radio propagation problems, the classical theory [Appleton and Chapman, 1932] suffices, especially when appropriate values of the effective collision frequency are used.

1.2 Measurement of absorption

The three principal techniques for measuring absorption are called the A1, A2, and A3 methods. These methods are described in detail in Rawer [1976] and Hunsucker [1991].

The A1 or *pulse-reflection method* consists of the measurement of the amplitude E_1, E_2, E_3, \dots , etc., of echoes reflected once, twice, etc., from the ionosphere. The method has been discussed by

Piggott et al. [1957]. The effective reflection coefficient ρ is given with just one ionospheric reflection by

$$\rho = \alpha E_1 h', \quad (2.a)$$

with two ionospheric reflections and one ground reflection by

$$\rho^2 \rho_g = 2\alpha E_2 h', \quad (2.b)$$

and so on; i.e.,

$$\rho^n \rho_g^{n-1} = n\alpha E_n h'. \quad (2.c)$$

Here ρ_g is the reflection coefficient of the ground, α is a calibration that depends on such factors as transmitter power and antenna gain, and h' is the group height that accounts for spatial diffraction of the wave. Hence

$$\rho \rho_g = \frac{2E_2}{E_1} = \frac{3E_3}{2E_2} = \dots \frac{(n+1)E_{n+1}}{nE_n}, \quad (2.d)$$

which eliminates the need to know α . The ground reflection coefficient can be estimated from the ground characteristics. Alternatively, $\rho \rho_g$ can be measured late at night when the absorption is small and $\rho \approx 1$. When ρ is known from eq. (2.d), substitution in eq. (2.a) gives α , which makes possible the measurement of absorption during the day, when only one echo may be present. The effects of distance attenuation on high frequency (HF) absorption measurements have been discussed by

Whitehead and Jones [1972]. The measurement of absorption is complicated by echo fading, with amplitude variations on order of $10:1$ over fading periods varying from a few seconds to tens of minutes. Amplitudes can be measured visually, photographically or, preferable, by digital sampling of the echo peak. Some additional problems arise because of the superposition of ordinary and extraordinary pulses, by partial reflections, by the dispersion of echoes, and by the presence of interfering signals.

The A2 or riometer (relative ionospheric opacity meter) technique [Little and Leinbach, 1959] measures the intensity of the wideband noise that impinges on the Earth from deep space. The equipment consists of a stable receiver, a suitable antenna, a recorder, and a noise-diode calibrator. The calibration is carried out periodically by switching from the antenna to the calibrator. The received noise power is a function of sidereal (star) time, since the antenna beam will sample the same part of sky each day, as the Earth rotates. The absorption is, therefore, the difference between the signal actually received and that which would have been received at the same sidereal time in the absence of absorption [Krishnaswamy et al., 1985]. In using this technique a compromise frequency has to be selected. On the one hand, one would prefer to use as high a frequency as possible to avoid deviative effects but on higher frequencies the galactic noise level decreases and the ionospheric absorption decreases. A typical frequency used with this technique is 30 MHz, with which absorption changes of about 0.1 dB can be measured. The problem of interference can be partially removed by recording the minimum signal received while the frequency is swept over a small bandwidth (but many receiver bandwidths). Another difficulty arises during solar radio bursts. Thus a polar-pointing antenna is desirable. For discussion of the limitations of the technique see Little [1957].

In the A3 method, the output of a continuous-wave (CW) transmitter is recorded. The method has the advantage of simplicity and sensitivity but it is incapable of discriminating between various echoes. This difficulty can be partially removed by a suitable choice of frequency; for example, near the gyro-frequency the contributions from the extraordinary ray wave and the higher order

echoes are small. In the HF bandwidth, a CW system can best be calibrated by assuming the night-time absorption to be zero. At low (LF) and middle (MF) frequencies, which are used for studying the night-time absorption, the system is calibrated using the ground wave. This technique is attractive since broadcast transmitters are available. Also, we are usually interested in changes of absorption so that accurate knowledge of the zero absorption level is not critical. For further discussion of this technique see Schtwentek [1966]. This technique is subdivided according to frequency bandwidth: A3(a) on frequencies higher than 2 MHz and A3(b) on frequencies lower than 2 MHz (see Rawer [1976] and Hunsucker [1991] for further details).

2. The ICEPAC formula

This section describes the equations used to calculate losses for one ionospheric reflection (1 hop) mode. The model is intended to cover the frequency bandwidth from 3 to 30 MHz. All modes which result from a complete electron density profile are covered, as well as sporadic-E modes. The theoretical background and method of measurements are given in the manual on absorption measurements [Rawer, 1976]. The equations described below are intended to be used on a worldwide basis, and are limited by the available worldwide prediction data base. The equations are based on the CCIR 252-2 [1970] loss equation using a philosophy that modifications are made only when measured values demand a change. The CCIR loss equation was primarily derived from F2 low-angle modes with operating frequencies not greater than the frequency of optimum traffic (FOT). For these conditions, there is no need to modify the equation.

Free-space losses result from the geometrical dispersion of energy as the radio wave propagates away from the transmitter. In ionospheric propagation, the incremental cross section of the ray bundle at the receiver depends upon the physical properties of the ionosphere medium and the geometry of the propagation path. Simplifying assumptions are made in the program so that transmission losses can be calculated in a practical manner. In the simplest model of sky-wave propagation, it is assumed that the Earth and the ionosphere are both flat and that the reflection is specular (mirror-like). For this type of propagation, the energy density diminishes as the inverse square of the ray-path distance [Piggott, 1953].

The absorption in the D-E regions of the ionospheric medium is usually the major loss (after free space) in radio wave propagation via the ionosphere. To describe this effect, Martyn's [1935] theorem relating vertical and oblique path and the quasi-longitudinal approximation to the absorption loss is used [Davies, 1990]:

$$L(f_{ob}) = L(f_v) \cos \varphi_0 \quad (3)$$

$$L(f_v) = C \int_{h_0}^{h(f_v)} \frac{\frac{N\nu}{\mu} dh}{(f_v + f_H)^2 + \left(\frac{\nu}{2\pi}\right)^2}, \quad (4)$$

where:

C = constant,

h_0 = height at bottom of ionospheric medium,

$h(f_v)$ = height of reflection for frequency f_v ,

$N = N(h)$ electron density profile,

$\nu = \nu(h)$ collision frequency,

μ = refractive index,

f_{ob} = oblique sounding frequency,

f_v = vertical sounding frequency,

f_H = gyrofrequency, (f_L , the longitudinal component of gyrofrequency, is usually put into equation)

φ_0 = angle of the Earth's normal to ray path at height h_0 .

Equation (4) can be put into the form [Davies, 1990]

$$L(f_v) = (\nu/c)(h' - h_p), \quad (5)$$

where:

ν = effective collision frequency

h' = virtual height of reflection,

h_p = phase height of reflection,

c = velocity of light.

Since h_p is bounded and h' is not, there is a strong dependence on frequency which cannot be explained simply by an inverse-frequency-squared law. The usual method of analysis has been to write equation (4) in the form (definition of the global absorption parameter A):

$$L(f_v) = \frac{A(f_v)}{(f_v + f_H)^2 + (\nu/2\pi)^2}, \quad (6)$$

Most analyses of absorption are based on estimated $A(f_v)$. This has sometimes been done by assuming non-deviative absorption only, and thereby trying to ignore the frequency dependence. When this is done with a small data base and no adjustment is made for the frequency dependence, the results can be misleading (and possibly inaccurate).

The CCIR absorption equation is based upon the U.S. Army Signal Radio Propagation Agency study [Laitinen and Haydon, 1962], with a modification for lower frequencies by Lucas and Haydon [1966]. The exponent of the frequency term $(f_v + f_H)$ and values for the terms $(\nu/2\pi)^2$ and $A(f_v)$ were determined by least square curve fitting. The oblique loss measurements were normalized to virtual loss measurements to give a standard comparison method, as well as to expand the data base, by the equation:

$$L(f_{ob}) = \frac{L(f_v)[(f_v + f_L)^2 + (\nu/2\pi)^2]}{(f_{ob} + f_H)^2 + (\nu/2\pi)^2} \sec \varphi_0. \quad (7)$$

The data used were for F-layer modes only. The fitted equation is the ICEPAC formula [Stewart, undated]:

$$L(f_{ob}) = \frac{677.2 \cdot I \cdot \sec \varphi_0}{(f_{ob} + f_L)^{1.98} + 10.2}. \quad (8)$$

That is, the averaged value of $A(f_v)$ is $677.2 \cdot I$, where [Stewart, undated]:

$$I = -0.04 + \exp(-2.937 + 0.8445 \cdot f_{oE}). \quad (9)$$

The formula for absorption index I is in terms of E plasma frequency f_{oE} which includes the variation in zenith angle and solar activity. There have been attempts to modify and replace equation (8) by independent researchers. Barghausen et al. [1969], as Schultz and Gallet [1970], used the methods described by Piggott [1953] on a smaller data base than used to derive equation (8), but did not add the frequency dependence (non-deviative absorption) suggested by Piggott [1953]: essentially the same as equation (5) is obtained, but for a parabolic E-layer. George [1971] has developed an absorption equation using an $A(f_v)$ which has an implicit $(h' - h_p)$ curve. The method of supplementing equation (8) described by Stewart [undated] was based upon area coverage and radar backscatter data and was developed in two steps, first to correct equation (8) for E-layer modes and then for frequency dependence (deviative absorption). It is essentially based upon the suggestion in Piggott [1953] and in George [1971], but with the advantage of having the electron density profiles available on a worldwide basis (see also Rawer [1976], Bibl et al., [1961], and Fejer [1961] for further discussion).

3. The IONORT-ISP system

Pezzopane et al. [2011] described how the joint utilization of autoscaled data such as the F_2 peak critical frequency f_oF_2 , the propagation factor $M(3000)F_2$, and the electron density profile coming from two reference stations (Rome and Gibilmanna), and the regional SIRMUP (Simplified Ionospheric Regional Model Updated) and global IRI (International Reference Ionosphere) models can provide a valid tool for obtaining a real-time three-dimensional (3-D) electron density mapping of the ionospheric medium. Preliminary results of the proposed 3-D model are shown by comparing the electron density profiles given by the model with the ones measured at three testing ionospheric stations (Athens, Roquetes, and San Vito).

Azzarone et al. [2012] described an useful software tool, called IONORT (IONOspheric Ray Tracing), for calculating a 3-D ray tracing of high frequency (HF) waves in the ionospheric medium. This tool runs under Windows operating systems and its user-friendly graphical interface facilitates both the numerical data input/output and the two/three-dimensional visualization of the ray path. In order to calculate the coordinates of the ray and the three components of the wave vector along the path as dependent variables, the core of the program solves a system of six first order differential equations, the group path being the independent variable of integration. IONORT uses a 3-D electron density specification of the ionosphere, as well as by geomagnetic field and neutral particles-electrons collision frequency models having validity in the area of interest.

The 3-D electron density representation of the ionosphere medium computed by the assimilative IRI-SIRMUP-P (ISP) model was tested using IONORT, the software application for calculating a 3-D ray-tracing for HF waves in the ionosphere [Settimi et al., 2013a]. A radio link was established between Rome (41.89° N, 12.48° E) in Italy, and Chania (35.51° N, 24.02° E) in Greece, within the ISP validity area, and for which oblique soundings are conducted. The ionospheric reference stations, from which the autoscaled f_oF_2 and $M(3000)F_2$ data and real-time vertical electron density profiles were assimilated by the ISP model, were Rome (41.89° N, 12.48° E) and Gibilmanna

(37.99°N, 14.02°E) in Italy, and Athens (37.98°N, 23.73°E) in Greece. IONORT was used, in conjunction with the ISP and the IRI 3-D electron density grids, to synthesize oblique ionograms. The comparison between synthesized and measured oblique ionograms, both in terms of the ionogram shape and the maximum usable frequency characterizing the radio path, demonstrates both that the ISP model can more accurately represent real conditions in the ionosphere than the IRI, and that the ray-tracing results computed by IONORT are reasonably reliable.

Many collision frequency models are available. For instance, the constant collision frequency, the exponential or tabular profiles [Jones and Stephenson, 1975]. To add other collision frequency models, the user must write a subroutine that will calculate the normalized frequency Z and its gradients ($\partial Z/\partial r$, $\partial Z/\partial \theta$, $\partial Z/\partial \phi$) as a function of position in spherical polar coordinates (r , θ , ϕ). The normalized frequency is defined as $Z = \nu/2\pi f$, where ν is the collision frequency between electrons and neutral air molecules and f is the wave frequency. The coordinates (r , θ , ϕ) refer to a computational coordinates system, which is not necessarily the same as geographic coordinates. In particular, the ionospheric dipole model of the Earth's magnetic field uses the axis of computational coordinate system as the axis for dipole field. When using this dipole model, the computational coordinate system is a geomagnetic coordinate system, and the Earth's magnetic field is defined in geomagnetic coordinates.

The IONORT-ISP system applies a frequency model consisting of a double exponential profile [Jones and Stephenson, 1975] (see Fig. 1)

$$\nu(h) = \nu_1 e^{-a_1(h-h_1)} + \nu_2 e^{-a_2(h-h_2)}, \quad (10)$$

where h is the height above the ground.

Specifying for the first exponential:

collision frequency at height h_1 , $\nu_1 = 3.65 \cdot 10^4$ collisions per second;

reference height, $h_1 = 100$ km;

exponential decrease of ν with height, $a_1=0.148 \text{ km}^{-1}$.

Specifying for the second exponential:

collision frequency at height h_2 , $\nu_1=30$ collisions per second;

reference height, $h_2=140 \text{ km}$;

exponential decrease of ν with height, $a_2=0.0183 \text{ km}^{-1}$.

3.1 The Oblique Sounding subroutine

Figs 2a,b show a flowchart of the IONORT-ISP (or IRI) system, calling in cascade the OBLIQUE SOUNDING and COMPLEIK subroutines (based on the COMPLEX EIKonal model) that produce respectively “SYNTHESIZED IONOGRAM.txt” and “L_COMPLEIK(dB) vs FREQUENCY(MHz).txt” as output files. The IONORT program runs on a file, named “GRIDPROFILES.txt”, which is the output of the ISP (or IRI) model, reproducing the 3-D electron density grid over the Mediterranean area on a chosen date (in the format dd/mmm/yyyy). IONORT reads an input file, named “RAY TRACING INPUT.ini”, to initialize the default inputs related to all the computational parameters needed by the ray-tracing algorithm, including the geographical position of the transmitter (TX), represented in a geocentric spherical coordinate system (R_{TX} , TH_{TX} , PH_{TX}) (R_{TX} in km, TH_{TX} and PH_{TX} in radians). The coordinates of the transmitter can be expressed in terms of its height $H_{TX}=R_{TX}-R_T$ (in km) from ground level, where $R_T=6371 \text{ km}$ is the averaged radius of the Earth, and in terms of latitude $LAT_{TX}=(180^\circ/\pi)(\pi/2-TH_{TX})$ and longitude $LON_{TX}=(180^\circ/\pi)PH_{TX}$ angles (in degrees). The program iteratively calculates a ray path for each nested loop cycle K relative to the frequency F_K , the elevation EL_K , and the azimuth AZ_K angles, specified by a frequency-step $F_K=F_K+F_{STEP}$, elevation-step $EL_K=EL_K+EL_{STEP}$, and azimuth-step $AZ_K=AZ_K+AZ_{STEP}$ procedure, with $K=START, \dots, END$ (see Fig. 2.a). The computing time footprint of a complete IONORT simulation can be estimated as $\Delta t \cdot N_{TOT}$, if Δt is assumed as the computing time of every loop cycle, where $N_{TOT}=N_F \cdot N_{EL} \cdot N_{AZ}$ is the total number of cycles, with

$N_F=1+[(F_{END}-F_{START})/F_{STEP}]$ the number of frequencies, $N_{EL}=1+[(EL_{END}-EL_{START})/EL_{STEP}]$ and $N_{AZ}=1+[(AZ_{END}-AZ_{START})/AZ_{STEP}]$ the number of elevations and azimuths respectively.

At the end of each K^{th} nested loop cycle, the OBLIQUE SOUNDING subroutine lists the parameters of ray-tracing output, i.e. the geographical position of the arrival point, still represented in the geocentric spherical coordinate system (R_K, TH_K, PH_K) , and also by the corresponding group delay time T_K (in ms). Moreover, the OBLIQUE SOUNDING subroutine reads an input file, named “RECEIVER.txt”, to read the geographical position of the receiver (RX), again represented in the geocentric spherical coordinate system, and the tolerated accuracy of the RX position, defined by the triplet of relative errors $(ERROR_R, ERROR_TH, ERROR_PH)$. These relative errors should not exceed the following reasonable upper limits: $ERROR_R \leq \varepsilon_{max}$, $ERROR_TH \leq 0.1\%$, $ERROR_PH \leq 0.1\%$. In fact, ε_{max} , i.e. the component 42 of vector \mathbf{W} represented in the input file “RAY TRACING INPUT.ini” used by IONORT (see Table 1 of Azzarone et al. [2012]), is defined as the maximum allowable relative error in any single step length for any of the equations being integrated. If $ERROR_TH$ and $ERROR_PH$ are less than 0.1%, then the arrival point is displaced from the receiver by not more than a few kilometers. Finally, the OBLIQUE SOUNDING subroutine compares the geographical positions of the arrival point and the receiver, assessing how close the arrival point is to the receiver. More precisely, OBLIQUE SOUNDING subroutine assumes that the target has been reached, or rather that the arrival point may be approximated with the receiver, when it falls within a circle centred on the receiver, of radius defined by the tolerated accuracies $(ERROR_R, ERROR_TH, ERROR_PH)$ of the RX position. Formally, the subroutine evaluates the following logical condition (see Fig. 2b):

$$(ABS((R_K-R_{RX})/R_{RX}) \leq ERROR_R).AND.$$

$$ABS((TH_K-TH_{RX})/TH_{RX}) \leq ERROR_TH).AND.ABS((PH_K-PH_{RX})/PH_{RX}) \leq ERROR_PH.$$

If this expression is false, then it means that the transmitter-receiver radio link has not been established at frequency F_K , elevation EL_K , and azimuth AZ_K . In this case, the OBLIQUE SOUNDING subroutine returns control to the IONORT system, so the K^{th} loop cycle is concluded while the next cycle starts. By contrast, the transmitter and receiver are radio linked and in this case, OBLIQUE SOUNDING updates the MUF variable ($MUF=F_K$) which, at the end of whole IONORT simulation, will coincide with the maximum usable frequency (MUF) of the radio link. Simultaneously, the subroutine stores in the final output file named “SYNTHESIZED IONOGRAM.txt”, the values of frequency F_K and group delay T_K that represent the K^{th} ionogram point $P_K=(F_K, T_K)$ relative to the K^{th} cycle giving rise to a transmitter-receiver radio link (see Fig. 2b). At the end of the complete simulation, the final output file will include all the N points, $P_{K1}=(F_{K1}, T_{K1})$, $P_{K2}=(F_{K2}, T_{K2})$, ... , $P_{KN}=(F_{KN}, T_{KN})$, composing the ionogram trace of the radio link. It is worth noting that the nested loop cycle in the azimuth transmission angle is optional. If the loop cycle in azimuth angle is not initiated, then the azimuth between the transmitter and the receiver becomes a fixed constant in the ray-tracing, i.e. $AZ_K=AZ_{\text{START}}=AZ_{\text{END}}$, which can be calculated simply by applying spherical trigonometry:

$$\text{tg}(AZ_{\text{START}})=\sin(TH_{\text{RX}})\sin(TH_{\text{TX}})\sin(PH_{\text{RX}}-PH_{\text{TX}})/[\cos(TH_{\text{RX}})-\cos(TH_{\text{TX}})\cos(\Delta_{\text{TX-RX}})],$$

where

$$\cos(\Delta_{\text{TX-RX}})=\cos(TH_{\text{RX}})\cos(TH_{\text{TX}})+\sin(TH_{\text{RX}})\sin(TH_{\text{TX}})\cos(PH_{\text{RX}}-PH_{\text{TX}}).$$

Omitting the azimuth cycle is convenient when possible, as the computing time footprint of the complete IONORT simulation is reduced to $\Delta t \cdot N_F \cdot N_{\text{EL}}$, with Δt the computing time for every cycle.

Fig. 3 plots the ordinary and extraordinary trace of the oblique ionograms, synthesized by the IONORT-ISP system, over the Rome-Chania radio link on 3 July 2011 at 17:00 UT, 4 July 2011 at

20:00 UT, 6 July 2011 at 12:00 UT, 7 July 2011 at 17:00 UT, 8 October 2011 at 06:15 UT, 9 October 2011 at 02:15 UT, 21 October 2010 at 12:00 UT, 29 May 2010 at 12:00 UT. WF (with field) indicates that the oblique ionograms were computed by taking the geomagnetic field into account. In the synthesized ionograms, with one ionospheric reflection (1 hop), the azimuth cycle is applied, with the elevation angle step set to 0.2° and the RX accuracy to 0.1%.

Each synthesized ionogram was computed without applying the collision frequency model (10), in order to plot the ionogram regardless of the absorption. Excluding the days 4 July and 9 October of the year 2011, respectively at the solar terminator on 20:00 UT and in the night time on 2:15 UT, for all the other cases, the oblique ionogram is composed by: 1) the trace corresponding to the ionospheric F1-F2 layers at high altitudes ($h > 150$ km) and characterized by a low absorption coefficient ($L \leq 20$ dB); 2) the trace corresponding to the E-layer at lower altitude ($90 \text{ km} < h \leq 150$ km) and with an higher absorption coefficient ($L > 20$ dB). Generally, the ionogram computed applying the collision frequency model (10) does not show the trace corresponding to the E-layer for the high absorption.

By the way, the E-layer must not be confused with the sporadic E-layer [Davis and Johnson, 2005]. IONORT runs on ISP, modelling just the normal and cyclic ionization properties of the ionospheric (non sporadic) E-layer, which is absent in the night time and occurs during the daylight hours, especially at the noon. Indeed, no conclusive theory has yet been formulated as to the origin of sporadic-E layer. As its name suggests, sporadic-E layer is an abnormal event, not the usual condition, but can happen at almost any time. Sporadic-E activity peaks predictably in the summertime in both hemispheres.

4. The Complex Eikonal model for the ionospheric D-layer

Settimi et al. [2013b] conducted a scientific review on the complex eikonal, extrapolating the research perspectives on the ionospheric ray-tracing and absorption.

As regards the scientific review, the eikonal equation is expressed, and some complex-valued solutions are defined corresponding to complex rays and caustics.

As regards the research perspectives, the authors continue the topics discussed by Bianchi et al. [2009], proposing a novelty with respect to the other referenced bibliography: indeed, the medium is assumed as dissipative. Under these conditions, suitable generalized complex eikonal and transport equations are derived.

Settimi et al. [2013c] conducted a scientific review on ionospheric absorption, extrapolating the research perspectives of a complex eikonal model for one-layer ionosphere.

As regards the scientific review, there is deduced a quasi-longitudinal (QL) approximation for non-deviative absorption which is more refined than the corresponding equation reported by Davies [1990].

As regards the research perspectives, there is analyzed in depth a complex eikonal model for one layer ionosphere, already discussed by Settimi et al. [2013b]. A simple formula is deduced for a simplified problem. A flat layering ionospheric medium is considered, without any horizontal gradient. The authors demonstrate that the QL non-deviative absorption coefficient according to the complex eikonal model is more accurate than Rawer's theory [1976] in the range of middle critical frequencies.

4.1 A simple formula for a simplified problem

The cited paper [Settimi et al., 2013b] proposed a new formula, useful to calculate the absorption due to the propagation across the ionospheric D-layer, which can be approximately

modelled by a linearized complex refractive index, covering a short range of heights between $h_1=50$ km and $h_2=90$ km.

According to Budden [1988], rocket techniques have evidenced that, in the daytime, the D-layer shows, almost as a rule, its maximum (and minimum) of electron density in the vicinity of 80 km (and 85 km). In authors opinion, this evidence is not so strong, and even if the refractive index $n_R(h)$ is not a monotonically decreasing function of height h along the D-E layers valley, anyway this is not a substantial correction: $n_R(h)$ can be linearized up to $h_2=90$ km about.

Settimi et al. [2013c], assuming the analytical continuity of complex eikonal model with the quasi-longitudinal (QL) approximation for non-deviative absorption, demonstrates the necessary and sufficient condition to equate the collision frequency deriving from the altitude profile adopted for complex refractive index [Settimi et al., 2013b],

$$n(h) = 1 \quad , \quad h < h_0, \quad (11.a)$$

$$\begin{cases} n(h) = n_R(h) + in_I(h) & , \quad h \geq h_0 \\ n_R(h) = n_0 + \alpha_R(h - h_0) \\ n_I(h) = \alpha_I \end{cases} \quad (11.b)$$

where h_0 is the height of the ionospheric bottom and $n_0=n(h_0)$ the refractive index across the ionosphere-neutral atmosphere boundary [in a first-order approximation, the boundary refractive index n_0 could be assumed as a real number slightly different from 1, i.e. in any case $n_0 \neq 1$, so that the refractive index $n(h)$ is a discontinuous function of height h , i.e. crossing $h=h_0$], to the variation of collision frequency with the altitude [Budden, 1988],

$$\nu(h) = \nu_{\max} \exp\left(-\frac{h-h_{\max}}{H}\right) \cong \nu_{\max} \left(1 - \frac{h-h_{\max}}{H}\right) \quad , \quad |h-h_{\max}| \ll H, \quad (12)$$

where H is the atmospheric scale height, ν_{\max} is a constant, i.e. $\nu_{\max}=\nu(h_{\max})$, and h_{\max} is the height corresponding to the maximum electron density N_{\max} , i.e. $N_{\max}=N(h_{\max})$ [the constant ν_{\max} is not the maximum collision frequency but the collision frequency at the “maximum height” h_{\max}].

The QL approximation for non-deviative absorption, as deduced by Settini et al. [2013c], is more refined than the corresponding equation reported by Davies [1990]; and, linearizing the involved equations, there are obtained the coefficients α_R and α_I as functions of the angular frequency ω , the collision frequency ν_{\max} , the scale height H , the mean magnetic gyro-frequency $\langle\omega_H\rangle$, and the height of the ionospheric bottom h_0 , the refractive index across the ionosphere-neutral atmosphere boundary $n_0=n(h_0)$:

$$\alpha_R = -\frac{1}{2} \frac{\frac{1-n_0^2}{H}}{1 + \frac{h_{\max}-h_0}{H}}, \quad (13.a)$$

$$\alpha_I = -\frac{(1-n_0)^2}{H} \frac{\nu_{\max}}{\omega \pm \langle\omega_H\rangle} \left[1 + \left(\frac{\nu_{\max}}{\omega \pm \langle\omega_H\rangle} \right)^2 \right]. \quad (13.b)$$

A reasonable hypothesis should be assumed for eq. (13): the boundary refractive index n_0 is a real number slightly less than 1, i.e. $n_0 < 1$, so that both the coefficients α_R and α_I are negative, i.e. $\alpha_R < 0$ and $\alpha_I < 0$.

Therefore, the linearized analytical profile for complex refractive index (13.b) can be re-arranged as:

$$n_R(h) \cong n_0 - \frac{1-n_0^2}{2} \frac{(h-h_0)/H}{1 + (h_{\max}-h_0)/H}, \quad h \gg h_0. \quad (14.a)$$

$$n_I = -\frac{(1-n_0)^2}{H} \frac{\nu_{\max}}{\omega \pm \langle\omega_H\rangle} \left[1 + \left(\frac{\nu_{\max}}{\omega \pm \langle\omega_H\rangle} \right)^2 \right]. \quad (14.b)$$

Just the reasonable hypothesis assumed below eq. (13) could imply the expected conclusions for eq. (14): the real refractive index $n_R(h)$ is a decreasing function of height h , while the imaginary refractive index n_I is negative, as substantially correct for any ionospheric profile of the D-layer [Budden, 1988].

Moreover, considering a vertical radio sounding with one ionospheric reflection (1 hop), once calculated the optical path $\gamma: h_1 \rightarrow h_2$ [Settimi et al., 2013c],

$$\Delta l_{12}^{(v)} = \int_{h_1}^{h_2} n_R(h) dh \cong (h_2 - h_1) \left[n_0 - \frac{1 - n_0^2}{4} \frac{(h_1 + h_2 - 2h_0)/H}{1 + (h_{\max} - h_0)/H} \right], \quad |h_2 - h_1| \ll h_0 < h_{\max}, \quad (15)$$

it results proportional to the integral absorption coefficient [Settimi et al., 2013c]:

$$\beta_{12}^{(v)} \cong 2 \frac{1 - n_0}{1 + n_0} \Delta l_{12}^{(v)} \left(1 + \frac{h_{\max} - h_0}{H} \right) \frac{\nu_{\max}}{c} \frac{\omega}{\omega + \langle \omega_H \rangle} \left[1 + \left(\frac{\nu_{\max}}{\omega + \langle \omega_H \rangle} \right)^2 \right]. \quad (16)$$

Note that the refractive index $n_0 = n(h_0)$ can be computationally assumed as $n_0 = 1 - \varepsilon_{\max}$ for any ray-tracing program, where ε_{\max} is defined as the maximum allowable relative error in single step length for any of the equations being integrated. To get a very accurate (but expensive) ray trace, it is possible use a small ε_{\max} (about 10^{-5} or 10^{-6}). For a cheap, approximate ray trace, a large ε_{\max} (10^{-3} or even 10^{-2}) should be used. For cases in which all the variables being integrated increase monotonically, the total relative error can be guaranteed to be less than ε_{\max} [Jones and Stephenson, 1975].

Instead, considering an oblique radio sounding with one ionospheric reflection (1 hop), the Martyn's [1935] absorption theorem assures that the integral absorption coefficient $\beta_{12}^{(ob)}$ of a wave

at angular frequency ω incident on a flat ionospheric medium with angle φ_0 is further dependent on the secant of φ_0 . It results a simple formula for a simplified problem [Settimi et al., 2013c]:

$$\begin{aligned} \beta_{12}^{(\text{ob})} \Big|_{\omega_{\text{ob}}=\omega} &= \beta_{12}^{(\text{v})} \Big|_{\omega_{\text{v}}=\omega \cos \varphi_0} \cos \varphi_0 = \\ &= 2 \frac{1-n_0}{1+n_0} \Delta I_{12}^{(\text{v})} \left(1 + \frac{h_{\text{max}} - h_0}{H} \right) \frac{v_{\text{max}}}{c} \frac{\omega}{\omega + \langle \omega_H \rangle} \left[\frac{1}{\sec \varphi_0} + \left(\frac{v_{\text{max}}}{\omega + \langle \omega_H \rangle} \right)^2 \sec \varphi_0 \right]. \end{aligned} \quad (17)$$

4.3 The COMPLEIK subroutine of the IONORT-ISP system

Implying the reader to be familiar with the mathematical symbols and their physical meaning, as introduced and discussed by Jones and Stephenson [1975], Azzarone et al., [2012], and Settimi et al., [2013a,b,c], let us report in Appendix A the FORTRAN code for the COMPLEIK (COMPLEx EIKonal) subroutine of the IONORT-ISP system (see Figs. 2a,b). The FORTRAN code is enriched by some comments. All variables are computed in the International System (SI) of units. Few auxiliary variables are employed, referring to the D-layer, which is modelled as one ionospheric layer between the heights $h1= 50$ km and $h2= 90$ km, being characterized by suitable mean values for the gravity acceleration g_MEAN , the absolute temperature T_MEAN , and the atmospheric scale height H . The auxiliary variables refer even to the whole ionospheric model, involving the refractive index n_h0 across the ionosphere-neutral atmosphere boundary $h0$, the mean value of magnetic gyro-frequency FH_MEAN , the height relative to the maximum electron density $hMAX$ and the corresponding collision frequency NU_hMAX . The COMPLEIK subroutine implements the complex eikonal model equations. The Martyn's [1935] absorption theorem is applied to transform a vertical to an oblique radio sounding with one ionospheric reflection (1 hop), calculating the secant SEC of incident angle $PH0$, the vertical optical path OPT_PATH , and the oblique absorption coefficient $L_COMPLEIK$, then reported in decibel units $L_COMPLEIK_dB$. The COMPLEIK subroutine stores, in the final output file so called "L_COMPLEIK(dB) vs FREQUENCY(MHz).txt", the values for frequency F_K and oblique absorption coefficient L_K which

corresponds to the K^{th} ionogram point $P_K=(F_K, T_K)$, relative to the K^{th} cycle giving rise to a transmitter-receiver radio link (see Fig. 2b). At the end of the complete simulation, the final output file will include all the N points, $P_{K1}=(F_{K1}, L_{K1})$, $P_{K2}=(F_{K2}, L_{K2})$, \dots , $P_{KN}=(F_{KN}, L_{KN})$, composing the absorption profile of radio link.

4.4 Analysis and Discussion

Let us consider a radio link, with one ionospheric reflection (1 hop), between the transmitter (TX), Rome, Italy ($lat_1=41.89^\circ\text{N}$, $lon_1=12.48^\circ\text{E}$) and the receiver (RX), Chania, Crete ($lat_2=35.51^\circ\text{N}$, $lon_2=24.02^\circ\text{E}$) stations. The IRI [2007] model allows computing the E plasma critical frequency f_{oE} on any date and time, relative to the midpoint M between TX and RX, $M=[lat_M=(lat_1+lat_2)/2=38.70^\circ\text{N}$, $lon_M=(lon_1+lon_2)/2=18.25^\circ\text{E}]$. The IRI parameter f_{oE} is an input of the ICEPAC formula [Stewart, undated], useful to calculate in a phenomenological way the non-deviative absorption coefficient, in quasi-longitudinal (QL) approximation [eqs. (8)-(9)]. Let use the IONORT program, simulating a three-dimensional (3-D) ray tracing of high frequency (HF) waves in the ionospheric medium, in conjunction with the ISP model, which generates a 3-D real-time electron density grid of the ionosphere over the Mediterranean area, in order to synthesize oblique ionograms of the radio link between Rome and Chania. The IONORT-ISP system allows computing the height h_{max} relative to the maximum electron density N_{max} , which is an input of the COMPLEIK subroutine (based in the COMPLEX EIKonal model), useful to calculate theoretically the QL non-deviative absorption coefficient [eqs. (15)-(17)].

Note that suitable operative conditions must hold to apply the theorems of Breit & Tuve and Martyn [Martyn, 1935][Davies, 1990]. Accordingly, the real oblique sounding between the stations of Rome and Chania can be transformed into a virtual oblique sounding, which oblique sounding, in turn, is reduced to a virtual vertical sounding along the vertical line of the midpoint M . Thus, the ICEPAC formula can be referred to the radio link Rome-Chania, once the IRI [2007] model has computed the critical frequency f_{oE} relative to the midpoint M . Indeed, the Breit & Tuve's and

Martyn's theorems should be applied during hours of the day in which the ionospheric medium is characterized by small horizontal gradients, when the azimuth angle of transmission is assumed to be a constant along the great circle path [Settimi et al., 2013a]. By the way, the COMPLEIK subroutine can be ideally used for a flat layering ionosphere, without any horizontal gradient, so characterized by an electron density profile dependent only on the altitude [Settimi et al., 2013b,c]. The complex eikonal model can be implemented more directly than the ICEPAC formula, since the height h_{\max} is computed numerically within the IONORT-ISP system, instead the critical frequency f_{oE} only in a phenomenological way, and even applying the IRI [2007] model.

Fig. 4 shows the semi-logarithmic plots of the percentage relative deviation [%], in the QL approximation, between the non-deviative integral absorption coefficients according to the complex eikonal model (COMPLEIK subroutine) [see eqs. (15)-(17)] and the ICEPAC formula [see eqs. (8) and (9)], as a function of the frequency f [MHz], for both the ordinary and extraordinary rays.

All the radio links Rome-Chania tested at frequencies within the HF bandwidth $6 \text{ MHz} \leq f \leq 18 \text{ MHz}$ show an absorption relative deviation that is minimum for the ordinary ray, assuming small values in the range $[10^{-1}\%, 10\%]$, and it is maximum for the extraordinary ray, assuming larger values in the range $[1\%, 10^2\%]$, especially at lowest frequencies ($6 \text{ MHz} \leq f \leq 10 \text{ MHz}$). Indeed, according to the complex eikonal model [Settimi et al., 2013b,c], the ionospheric D-layer can be approximately characterized by a linearized analytical profile of complex refractive index, covering a short range of heights between $h_1 = 50 \text{ km}$ and $h_2 = 90 \text{ km}$. Note that the complex eikonal model is not numerically reliable for the low critical frequencies, as the integral absorption coefficient implies tacitly that not so low altitudes are involved [Settimi et al., 2013c]. At any rate, in the HF bandwidth, the non-deviative absorption is mostly limited to the D-layer [Rawer, 1976]. The ordinary ray is partially absorbed just during the propagation across the D-layer, thus its QL non-deviative absorption coefficient is rightly calculated by the COMPLEIK subroutine. Instead, according to the ISP model [Pezzopane et al., 2011], the ionospheric medium is characterized by an electron density profile defined on the heights range $81 \text{ km} \leq h < 378 \text{ km}$, de facto excluding the D-

layer ($50 \text{ km} \leq h \leq 90 \text{ km}$), though including the E-layer ($90 \text{ km} < h \leq 150 \text{ km}$). Note that if the extraordinary ray propagates at lowest radio frequencies, then the Appleton-Hartree dispersion formula may not be adequate in the D-E layers [Davies, 1990]. Within the QL approximation, the lower the radio frequencies, the more the extraordinary ray is reflected at bottom altitudes compared to the ordinary ray. At any rate, for HF sounding, there are two distinct height ranges from which appreciable contributions are to be expected: in the D-layer, where the non-deviative formulae may be used, and in the vicinity of E-layer reflection height, where the deviative term must be taken into account. The situation is similar for higher frequencies in the HF bandwidth which penetrate the E-layer and are reflected from the F-layer. In this case, some non-deviative absorption on the E-layer may also be important [Rawer, 1976]. The extraordinary ray spends a longer time in the E-layer than in the D-layer, feeling both the deviative and non-deviative absorptions due to the E-layer as well as the non-deviative absorption due to the D-layer. Thus, its QL non-deviative absorption coefficient is more correctly calculated by the ICEPAC formula, modelling even the E-layer, than the COMPLEIK subroutine, which models only the D-layer.

Fig. 5 shows the semi-logarithmic plots of the QL approximation for non-deviative absorption coefficients [dB], according to the complex eikonal model (COMPLEIK subroutine) [see eqs. (15)-(17)] and the ICEPAC formula [see eqs. (8) and (9)], as a function of the frequency f [MHz], for both the ordinary and extraordinary rays.

Excluding the days 4 July and 9 October of the year 2011, respectively at the solar terminator on 20:00 UT and in the night time on 2:15 UT, for all the other cases, the QL non-deviative absorption profile is composed by: 1) a pair of ordinary and extraordinary traces corresponding to the ionospheric F1-F2 layers at high altitudes ($h > 150 \text{ km}$) and characterized by a low absorption coefficient ($L \leq 20 \text{ dB}$); 2) another pair of ordinary and extraordinary traces corresponding to the E-layer at bottom altitude, $90 \text{ km} < h \leq 150 \text{ km}$, and with an higher absorption coefficient ($L > 20 \text{ dB}$)(see Fig. 3). Compared to the ICEPAC formula, the COMPLEIK subroutine provides the best results for the low absorption traces; indeed, the ICEPAC-COMPLEIK fitting is optimal for both

the ordinary ray, in the whole HF bandwidth, and the extraordinary ray, in a wide neighbourhood of the maximum usable frequency (MUF). Instead, the COMPLEIK subroutine provides the worst results for the high absorption traces: indeed, the ICEPAC-COMPLEIK fitting tends to worsen, not only for the ordinary ray but much more for the extraordinary ray, especially in the lowest frequencies of the HF bandwidth. Note that if the extraordinary ray propagates at lowest radio frequencies, then the ICEPAC formula may not be adequate in the D-E layers [Stewart, undated]. The complex eikonal model is not numerically reliable for the highest critical frequencies, when the linearized complex refractive index fails, having to be replaced by a parabolic or even cubic profile [Settimi et al., 2013c]. At any rate, these results should be improved by extending the complex eikonal model to higher altitudes in order to include even the E-layer besides the D-layer. It could not be worth the effort, since other equations would result for the non-deviative absorption coefficient, much less handy than eqs. (15)-(17), and possibly involving numerical problems due to spurious oscillations.

In this paper, the complex eikonal model [Settimi et al., 2013b,c] has been used in conjunction with the discrete assimilative ISP grid profiles of the ionospheric electron density over the Mediterranean area [Pezzopane et al., 2011]. As main results, the COMPLEIK subroutine, characterizing only the D-layer, has been joined up to the IONORT-ISP system [Settimi et al., 2013a], which simulates the 3-D ray-tracing for HF waves in the ionospheric medium, de facto excluding the D-layer, though including the E-layer, in order to allow calculating the QL non-deviative absorption coefficient due to the radio propagation in the D-layer (and E-layer). Then, compared to the ICEPAC formula, the non-deviative absorption profiles corresponding to the COMPLEIK subroutine are reliable for all the ordinary and extraordinary rays effectively radio linking Rome and Chania. Indeed, the transmitting frequencies, generally tuned in the neighborhood of the MUF, are partially absorbed in the D-E layers and are propagating in the F1-F2 layers. Instead, the absorption profiles corresponding to the COMPLEIK subroutine lose their reliability just for all the rays that cannot establish the radio link Rome-Chania. Indeed, these non-transmitting

frequencies, especially if tuned away from the MUF, are fully absorbed by the D-E layers and so cannot propagate in the F1-F2 layers.

5. Conclusions

The IONORT (IONOspheric Ray-Tracing) program [Azzarone et al., 2012], which simulates the three-dimensional (3-D) ray-tracing for high frequencies (HF) waves in the ionospheric medium, runs on the assimilative ISP (IRI-SIRMUP-P) discrete model over the Mediterranean area [Pezzopane et al., 2011]. Even if the IONORT-ISP system [Settimi et al., 2013a] involves a model of collision frequency, anyway it is not enabled to compute the absorption coefficient, neither non-deviative nor deviative. After all, according to the ISP model, the ionosphere is characterized by an electron density profile defined on the heights range $81 \text{ km} \leq h < 378 \text{ km}$, de facto excluding the D-layer ($50 \text{ km} \leq h \leq 90 \text{ km}$), though including the E-layer ($90 \text{ km} < h \leq 150 \text{ km}$).

As main outcome of the paper, now the IONORT program includes a so called COMPLEIK (COMPLex EIKonal) subroutine. Thus, the IONORT-ISP-COMPLEIK system is enabled to compute, in quasi-longitudinal (QL) approximation, just the non-deviative absorption coefficient, due to the propagation across the ionospheric D-layer. Indeed, the COMPLEIK subroutine encodes the complex eikonal equations [Settimi et al., 2013b,c], which model the D-layer approximately by a linearized analytical profile of complex refractive index, covering the short range of heights between $h_1 = 50 \text{ km}$ and $h_2 = 90 \text{ km}$.

In this paper, the simple COMPLEIK algorithm has been compared to the more elaborate semi-empirical ICEPAC formula [Stewart, undated], which refers to various phenomenological parameters such as the critical frequency of E-layer. COMPLEIK is reliable just like ICEPAC, with the advantage of being implemented more directly. Indeed, the complex eikonal model depends only on some parameters of the electron density profile, which are numerically calculable, such as the maximum height.

In a forthcoming paper, the IONORT-ISP-COMPLEIK system will be enabled to compute, in quasi-longitudinal (QL) [or quasi-transverse (QT)] approximation, the deviative (or non-deviative) absorption coefficient, due to the propagation (or reflection) within the E-layer. Indeed, the complex

eikonal theory may be extended to higher altitudes in order to join up the discrete model (ISP) to the analytical model (COMPLEIK) of E-layer, replacing the linearized complex refractive index with a parabolic or even cubic profile.

Appendix A

```
C*****
C COMPLETEIK: The algorithm of COMPLETEIK subroutine
C simulates the non-deviative absorption by ionospheric collisions,
C based on the following simplifying hypotheses:
C quasi-longitudinal (QL) propagation for the ray-tracing;
C a linear profile of electron density for the D-layer;
C a dipole model for the geomagnetic field;
C an exponential model for the collision frequency.
C*****
SUBROUTINE COMPLETEIK COMPLEIK01
C*****
C*****
C SEE: Jones and Stephenson [1975]; Azzarone et al. [2012].
C*****
COMMON R(20),T,STP,DRDT(20),N2 COMPLEIK02
COMMON /CONST/ PI,PIT2,PID2,DEGS,RAD,K,C,LOGTEN COMPLEIK03
COMMON /WW/ ID,W0,W(400) COMPLEIK04
COMMON /RK/ N,STEP,MODE,E1MAX,E1MIN,E2MAX,E2MIN,FACT,RSTART COMPLEIK05
COMMON /XX/ MODX(2),X,PXPR,PXPTH,PXPPH,PXPT,hMAX COMPLEIK06
C*****
C*****
C Optical ray incident on a flat ionospheric medium
C forming the secant angle PH0
C*****
COMMON /PH0/ PH0 COMPLEIK07
C*****
C*****
C COMPLETEIK OUTPUT: Absorption coefficient [dB].
C*****
COMMON /L_COMPLEIK_dB/ L_COMPLEIK_dB COMPLEIK08
C*****
C*****
C SEE: Jones and Stephenson [1975]; Azzarone et al. [2012].
C*****
COMPLEX N2 COMPLEIK09
REAL LOGTEN COMPLEIK10
REAL NU1 , NU2 COMPLEIK11
EQUIVALENCE (RAY,W(1)) , (EARTH, W(2)) , (F,W(6)) , COMPLEIK12
> (FH,W(201)) , COMPLEIK13
> (NU1,W(251)) , (z1,W(252)) , (A1,W(253)) , COMPLEIK14
> (NU2,W(254)) , (z2,W(255)) , (A2,W(256)) COMPLEIK15
C*****
C*****
C SEE: Settini et el. [2013b,c].
C*****
REAL K1 , K2 , K3 , K4 , K5 COMPLEIK16
```

```

REAL kB ,
> m , mH , mp , me ,
> h1 , T1 ,
> h2 , T2 ,
> g0, g_MEAN ,
> T_MEAN , H ,
> ONE , h0 , n_h0,
> FH_MEAN , NU_hMAX ,
> ANGLE , SEC ,
> OPT_PATH , L_COMPLEIK , L_COMPLEIK_dB
DATA K1/0.0053024/ , K2/0.0000058/ ,
> K3/0.000003086/ , K4/0.2857/ , K5/0.770982/
DATA h1/50./ , T1/273./ ,
> h2/90./ , T2/187./
C*****
C*****
C INTERNATIONAL SYSTEM (SI) OF UNITS
C*****
C*****
C Boltzmann's constant
C*****
kB=1.3806488*1E-23
C*****
C*****
C Electron, proton, hydrogen atom mass and atmosphere mean molecular mass
C*****
me=9.10938291*1E-31
mp=1.672621777*1E-27
mH=me+mp
m=29.0*mH
C*****
C*****
C IONOSPHERIC D-LAYER BETWEEN THE HEIGHTS  $h1=50$  km AND  $h2=90$  km
C*****
C*****
C International Gravity Formula (IGF) 1967:
C Mean value of gravity acceleration.
C*****
g0=9.780327
g_MEAN=g0*(1.+(K1-K2)/2)-K3*1.E3*(h1+h2)/2
C*****
C*****
C Mean value of absolute temperature
C*****
T_MEAN=T2*(1.-K4*((m*g_MEAN)/(2.*kB*T2))*1.E3*(h1-h2))
C*****
C*****
C Atmospheric scale height
C*****
H=1.E-3*((kB*T_MEAN)/(m*g_MEAN))
C*****

```

COMPLEIK17
 COMPLEIK18
 COMPLEIK19
 COMPLEIK20
 COMPLEIK21
 COMPLEIK22
 COMPLEIK23
 COMPLEIK24
 COMPLEIK25
 COMPLEIK26
 COMPLEIK27
 COMPLEIK28
 COMPLEIK29
 COMPLEIK30

 COMPLEIK31

 COMPLEIK32
 COMPLEIK33
 COMPLEIK34
 COMPLEIK35

 COMPLEIK36
 COMPLEIK37

 COMPLEIK38

 COMPLEIK39

```

C*****
C In order to smooth out numerical problems due to spurious oscillations,
C the constant ONE can be computationally assumed as
C ONE=1.-E1MAX for any ray-tracing program,
C where E1MAX is defined as
C the maximum allowable relative error in single step length
C for any of the equations being integrated.
C*****
ONE=1. COMPLEIK40
ONE=ONE-E1MAX COMPLEIK41
C*****
C*****
C SEE: Pezzopane et al. [2011]; Settini et al. [2013b,c].
C*****
C*****
C h0:
C the height of the ionospheric bottom.
C According to the ISP (IRI-SIRMUP-P) model,
C the ionosphere is characterized by an electron density profile
C defined on the heights range  $81\text{ km} \leq h \leq 378\text{ km}$ ,
C de facto excluding the D-layer ( $50\text{ km} \leq h \leq 90\text{ km}$ ),
C though including the E-layer ( $90\text{ km} < h \leq 150\text{ km}$ ).
C According to the COMPLEIK (COMPLEx EIKonal) model,
C the D-layer can be approximately characterized
C by a linearized analytical profile of complex refractive index,
C covering a short range of heights between  $h1 = 50\text{ km}$  and  $h2 = 90\text{ km}$ .
C The COMPLEIK subroutine
C has been joined to the IONORT-ISP system
C for including the whole D-layer:
C thus, the height of ionospheric bottom h0 must be below  $h1 = 50\text{ km}$ .
C*****
h0=h1 COMPLEIK42
C*****
C*****
C n_h0:
C the refractive index across the ionosphere-neutral atmosphere boundary.
C In a first-order approximation,
C the boundary refractive index n0 could be assumed
C as a real number slightly less than 1, i.e.  $n0 < 1$ ,
C so that the real refractive index nR(h)
C is a decreasing function of height h,
C while the imaginary refractive index nI is negative,
C as substantially correct for any ionospheric profile of the D-layer
C [Budden, 1988].
C*****
n_h0=ONE COMPLEIK43
C*****
C*****
C Mean value of magnetic gyro-frequency
C*****
FH_MEAN=K5*FH* COMPLEIK44

```



```

>(((EARTH**3)*(h1+h2+2*EARTH))/
COMPLEIK45
>(((h1+EARTH)**2)*(h2+EARTH)**2)))
COMPLEIK46
C*****
C*****
C Collision frequency model
C consisting of a double exponential profile [Jones and Stephenson, 1975];
C hMAX is the height relative to the maximum electron density,
C which can be numerically calculated.
C*****
NU_hMAX=NU1*EXP(-A1*(hMAX-z1))+NU2*EXP(-A2*(hMAX-z2))
COMPLEIK47
C*****
C*****
C APPLYING MARTYN'S ABSORPTION THEOREM [MARTYN, 1935]:
C*****
SEC=1/COS(PH0)
COMPLEIK48
C*****
C*****
C FROM A VERTICAL RADIO SOUNDING WITH ONE IONOSPHERIC REFLECTION
C*****
C*****
C The optical path  $\gamma$ :  $h1 \rightarrow h2$ 
C results proportional to the vertical absorption coefficient
C*****
OPT_PATH=(h2-h1)*
COMPLEIK49
> (n_h0-((1.-n_h0**2)/4)*
COMPLEIK50
> (((h2+h1-2.*h0)/H)/(1.+((hMAX-h0)/H))))
COMPLEIK51
C*****
C*****
C TO AN OBLIQUE RADIO SOUNDING WITH ONE IONOSPHERIC REFLECTION
C*****
C*****
C The oblique absorption coefficient
C of a wave incident on a flat ionosphere forming the angle PH0
C is further dependent on the secant of PH0
C*****
L_COMPLEIK=2.*((1.-n_h0)/(1.+n_h0))*
COMPLEIK52
> OPT_PATH*(1.+((hMAX-h0)/H))*
COMPLEIK53
> (NU_hMAX/C)*(F/(F+RAY*FH_MEAN))*
COMPLEIK54
> ((1./SEC)+SEC*((1.E6*NU_hMAX)/(PIT2*(F+RAY*FH_MEAN)))**2)
COMPLEIK55
C*****
C*****
C Reporting the absorption coefficient L_COMPLEIK in decibel units [dB]
C*****
L_COMPLEIK_dB=-(20./LOGTEN)*L_COMPLEIK
COMPLEIK56
C*****
C*****
C COMPLEIK OUTPUT: Absorption coefficient [dB].
C*****
OPEN(6,FILE='L_COMPLEIK(dB) vs FREQUENCY(MHz).txt',
COMPLEIK57
>STATUS='unknown')
COMPLEIK58
WRITE (6,1) F,ABS(L_COMPLEIK_dB)
COMPLEIK59

```

1 FORMAT (2(2X,F20.10))	COMPLEIK60
C CLOSE (6)	COMPLEIK61
C*****	
RETURN	COMPLEIK62
END	COMPLEIK63

References

- Appleton, E. V. and F. W. Chapman (1932). The collisional friction experienced by vibrating electrons in ionized air, *Proc. Phys. Soc., London*, 44 (243), 246–254.
- Azzarone, A., C. Bianchi, M. Pezzopane, M. Pietrella, C. Scotto and A. Settimi (2012). IONORT: A Windows software tool to calculate the HF ray tracing in the ionosphere, *Comp. Geosc.*, 42, 57-63, doi:10.1016/j.cageo.2012.02.008.
- Barghausen, A. F., J. W. Finney, L. L. Proctor and L. D. Schultz (1969). Predicting long-term operational parameters of high-frequency sky-wave telecommunication systems, ESSA Technical Report ERL 110-ITS78, Washington, U.S.A..
- Bianchi, S., U. Sciacca and A. Settimi (2009). Teoria della propagazione radio nei mezzi disomogenei (Metodo dell'iconale), *Quaderni di Geofisica*, 75, 14 pp. [in Italian].
- Bibl, K., A. Paul and K. Rawer (1961). Absorption in the D and E regions and its time variation, *J. Atmos. Terr. Phys.*, 23, 244-259.
- Budden, K. G. (1965). Effect of electron collisions on the formulas of magnetoionic theory, *Radio Science*, 69D(2), 191-211.
- Budden, K. G. (1988). The propagation of the radio wave. Cambridge University Press, Cambridge, UK, 688 pp.

- CCIR (1970). CCIR interim method for estimating sky-wave field strength and transmission loss at frequencies between the approximate limits of 2 and 30 MHz, International Radio Consultative Committee (CCIR) Report 252-2, International Telecommunication Union (ITU), Geneva, Switzerland, 202 pp..
- Davis, C. J., and C. G. Johnson (2005). Lightning-induced intensification of the ionospheric sporadic E layer, *Nature*, 435, 799-801, doi:10.1038/nature03638.
- Davies, K. (1990). *Ionospheric Radio*, Peter Peregrinus Ltd. (ed.), London, UK, 508 pp.
- Fejer, J. A. (1961). The absorption of short radio waves in the ionospheric D and E regions, *J. Atmos. Terr. Phys.*, 23, 260-274.
- George, P. L. (1971). The calculation of ionospheric absorption in HF radio propagation prediction, WRE-Technical Note-A207(AP), Department of Supply, Weapons Research Establishment, South Australia.
- Hunsucker, R. D. (1991). Radio techniques for probing the terrestrial ionosphere, in *Series on physics and chemistry in space*, Vol. 22, L. Lanzerotti (Editor), Springer-Verlag, New York, USA, 293 pp..
- International Reference Ionosphere (IRI), (2007). Virtual Ionosphere, Thermosphere, Mesosphere Observatory (VITMO) (http://ccmc.gsfc.nasa.gov/modelweb/models/iri_vitmo.php).
- Jones, R. M. and J. J. Stephenson (1975). A versatile three-dimensional ray tracing computer program for radio waves in the ionosphere. OT Report, 75-76, U. S. Department of

Commerce, Office of Telecommunication, U. S. Government Printing Office, Washington, USA, 185 pp.

Krishnaswamy, S., D. L. Detrick and T. J. Rosenberg (1985). The inflection point method of determining riometer quiet day curves, *Radio Science*, 20(1), 123-136.

Laitinen, P. O. and G. W. Haydon (1962). Analysis and prediction of sky-wave field intensities in the high frequency band, U. S. Army Signal Radio Propagation Agency Tech. Report No. 9, RPN 203, Monmouth, New Jersey, U.S.A., 134 pp..

Little, C. G. (1957). The measurement of ionospheric absorption using extra-terrestrial radio waves, in *Annals of the IGY*, Vol. III, W.J. G. Beyon and G. M. Brown (Editors), Pergamon, London, UK, 207 pp.

Little, C. G. and H. Leinbach (1959). The riometer – a device for the continuous measurement of ionospheric absorption, *Proc. IRE*, 47(2), 315-320.

Lucas, D. L. and G. W. Haydon (1966). MUF-FOT predictions by electronic computers, National Bureau Of Standards (NBS) Report 6789, U. S. Department of Commerce, Boulder, Colorado, U.S.A., 62 p.

Martyn, D. F. (1935). The propagation of medium radio waves in the ionosphere, *Proc. Phys. Soc.*, 47 (2), 323-339.

Pezzopane, M., Pietrella, M., Pignatelli, A., Zolesi, B., Cander, L.R., 2011. Assimilation of autoscaled data and regional and local ionospheric models as input sources for real-time 3-D

International Reference Ionosphere modeling, Radio Science, 46, RS5009, doi:10.1029/2011RS004697.

Phelps, A. V. and J. L. Pack (1959). Electron collision frequencies in nitrogen and in the lower ionosphere, Phys. Rev. Letter 3(7), 340-342.

Piggott, W. R. (1953). The reflection and absorption of radio waves in the ionosphere, Proc. IEEE, Part III: Radio and Communication Engineering, 100, No. 64, pp. 61-72.

Piggott, W. R., W. J. G. Beynon, G. M. Brown and C. G. Little (1957). The measurement of ionospheric absorption, in Annals of the IGY, Vol. III, Part II, Pergamon Press, London, UK, 173 pp.

Rawer, K. (1976). Manual on ionospheric absorption measurements. Report UAG – 57, edited by K. Raver, published by World Data Center A for Solar-Terrestrial Physics, NOAA, Boulder, Colorado, USA, and printed by U.S. Department of Commerce National Oceanic and Atmospheric Administration Environmental Data Service, Asheville, North Carolina, USA, 202 pp.

Schultz, L. D. and R. M. Gallet (1970). A survey and analysis of normal ionospheric absorption measurements obtained from pulse reflections, ESSA Professional Paper 4, U.S. Environmental Science Services Administration, Washington, U.S.A., 35 pp..

Schwentek, H. (1966). The determination of absorption in the ionosphere by recording the field strength of a distant transmitter, Ann. Geophys., 22, p. 276.

- Sen, H. K. and A. A. Wyller (1960). On the Generalization of the Appleton-Hartree Magnetoionic Formulas, *J. Geophys. Res.*, 65 (12), 3931-3950.
- Settimi, A., M. Pezzopane, B. Zolesi, M. Pietrella, C. Bianchi, C. Scotto, E. Zuccheretti, J. Makris (2013a). Testing the IONORT-ISP system: a comparison between synthesized and measured oblique ionograms, *Radio Science*, 48, RDS20018 [13 pages], doi: 10.1002/rds.20018.
- Settimi, A., U. Sciacca and C. Bianchi (2013b). Scientific review on the Complex Eikonal, and research perspectives on the Ionospheric Ray-tracing and Absorption, *Quaderni di Geofisica*, 112, 29 pp..
- Settimi, A., A. Ippolito, C. Cesaroni and C. Scotto (2013c). Scientific review on the ionospheric absorption and research perspectives of a Complex Eikonal model for one-layer Ionosphere, re-submitted for publication on *Ann. Geophys.* (16/10/2013).
- Stewart, F. G. (undated). Ionospheric Communications Enhanced Profile Analysis & Circuit (ICEPAC) Prediction Program, Technical Manual, 91 pp
(http://elbert.its.bldrdoc.gov/hf_prop/manuals/icepac_tech_manual.pdf).
- Thrane, E. V. and W. R. Piggott (1966). The Collision Frequency in the D- and E-Regions of the ionosphere, *J. Atmos. Terr. Phys.*, 28(8), 721-737.
- Whitehead, J. D. and T. B. Jones (1972). Distance attenuation effects in the absorption of vertically incident radio waves, *J. Atmos. Terr. Phys.*, 34(1), 165-169.

Figure captions

Figure 1. Semi-logarithmic plot of the collision frequency ν , in units of s^{-1} , as function of the height above ground h , in the range [50.0 km, 250.0 km][see eq. (10)].

Figure 2a. Flowchart of the IONORT-ISP (or IRI) system, calling in cascade the OBLIQUE SOUNDING and COMPLEIK (COMPLex EIKonal) subroutines that produce respectively “SYNTHESIZED IONOGRAM.txt” and “L_COMPLEIK(dB) vs FREQUENCY(MHz).txt” as output files. The iterative procedure consists of a nested loop cycle with frequency, elevation and azimuth steps.

Figure 2b. Exploded details of the logical box shown at the centre of Fig. 2a containing the IONORT program and both the OBLIQUE SOUNDING and COMPLEIK subroutines.

Figure 3. Ordinary and extraordinary trace of the oblique ionograms, synthesized by the IONORT-ISP system, over the Rome-Chania radio link on 3 July 2011 at 17:00 UT, 4 July 2011 at 20:00 UT, 6 July 2011 at 12:00 UT, 7 July 2011 at 17:00 UT, 8 October 2011 at 06:15 UT, 9 October 2011 at 02:15 UT, 21 October 2010 at 12:00 UT, 29 May 2010 at 12:00 UT. WF (with field) indicates that the oblique ionograms were computed by taking the geomagnetic field into account. In the synthesized ionograms, with one ionospheric reflection (1 hop), the azimuth cycle is applied, with the elevation angle step set to 0.2° and the receiver (RX) accuracy to 0.1%.

Figure 4. Refer to the caption of Fig. 3. Semi-logarithmic plots of the percentage relative deviation [%], in the quasi-longitudinal (QL) approximation, between the non-deviative integral absorption coefficients according to the complex eikonal model (COMPLEIK subroutine) [see eqs. (15)-(17)]

and the ICEPAC formula [see eqs. (8) and (9)], as a function of the frequency f [MHz], for both the ordinary and extraordinary rays.

Figure 5. Refer to the caption of Fig. 3. Semi-logarithmic plots of the QL approximation for non-deviative absorption coefficients [dB], according to the complex eikonal model (COMPLEIK subroutine) [see eqs. (15)-(17)] and the ICEPAC formula [see eqs. (8) and (9)], as a function of the frequency f [MHz], for both the ordinary and extraordinary rays.